EKPO DEBORAH JOSEPH COMPUTER ENGINEERING
17/ENG02/019

ENG 381 – ASSIGNMENT

The dynamic model of a body in motion performing damped force vibrations is as in Equation 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad \text{---} \quad (1)$$

Given that when $t = 0$, $x = 0.1$ and $dx/dt = 0$.

a Using Auxiliary Equation method, obtain the solution of the model in form of an expression having x as a function of t,

b With the aid of MATLAB m-file program, plot the diagram relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and

c Write the steady-state solution of the model in form of $x = k\sin(t + a)$

Solution

a $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$m^2 + 5m + 6m = 0$

$m^2 + 3m + 2m + 6 = 0$

$(m+3)(m+2) = 0$

$m = -3, -2$

Auxiliary Equation $= Ae^{-2t} + Be^{-3t}$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t + (-5C\sin t + 5D\cos t) + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

Taking the coefficient of cos, and sin on both sides

Cost: $-C + 5D + 6C = 1$

$5C + 5D = 1$

~~$5C = 1 - 5D$~~

$5C = 1 - 5D$

$C = \frac{1-5D}{5}$

$\sin t$: $-D - 5C + 6D = 0$

$-5C + 5D = 0$

$-5\left(\frac{1-5D}{5}\right) + 5D = 0$

$\frac{-5 + 25D}{5} + 5D = 0$

$-1 + 5D + 5D = 0$

$-1 + 10D = 0$

$10D = 1$

$D = 1/10 = 0.1$

$C = \frac{1-5D}{5}$

$= \frac{1 - 5(0.1)}{5} = \frac{1 - 0.5}{5} = \frac{0.5}{5} = 0.1$

$x = Ae^{-2t} + Be^{-3t} + 0.1\cos t + 0.1\sin t$

Substituting values of x and t

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 0.1\cos(0) + 0.1\sin(0)$

$0.1 = A + B + 0.1$

$0.1 - 0.1 = A + B$

$A + B = 0$ ---②

$\therefore A = -B$ ---①

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 0.1\sin t + 0.1\cos t$

Substituting values for dx/dt and t

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - 0.1\sin(0) + 0.1\cos(0)$

$0 = -2A - 3B + 0.1$

Recall eqn ① that $A = -B$; substituting

$0 = -2(-B) - 3B + 0.1$

$0 = 2B - 3B + 0.1$

$0.1 = K\cos a$

$K^2\sin^2 a + K^2\cos^2 a = 0.1 + 0.1$

$K^2(\sin^2 a + \cos^2 a) = 2/100$; $\sin^2 a + \cos^2 a = 1$

$K^2 = 2/100$

$K = \frac{\sqrt{2}}{10}$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1}$

$\tan a = 1$

$a = \tan^{-1}(1)$

$a = \pi/4$ or 45

$x_{ss} = \frac{\sqrt{2}}{10}\sin(t + \pi/4)$

b MATLAB

Command window

clear